J. Math. Kyoto Univ.
2-1 (1962) 55-60.

On the a priori estimate for solutions of the Cauchy problem for some non-linear wave equations

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(Received Aug. 30, 1962)

For the global Cauchy problem of wave equation, the existence of an a priori estimate of the solution is very useful as we have shown recently in another report [1] [2] for one special type of the non-linear wave equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + f(u)\frac{\partial u}{\partial t} + g(u)$$

under relatively weak conditions.

Here, we note that a priori estimate is also obtained for wave equation of a little different type with more than one space dimension which is identical to the equation treated by Konrad Jorgens [3] in the case of 3 dimension and without damping term.

At first we shall treat the case in which the space dimension is 2. Our Cauchy problem is the following: Find the solution of the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial t^2} + f\left(\frac{\partial u}{\partial t}\right) + g(u) \tag{1}$$

satisfying the following initial conditions.

$$\begin{cases} u(x, y, 0) = u_0(x, y) \\ u_t(x, y, 0) = u_1(x, y) \end{cases} \tag{2}$$

where $u_0(x, y)$ belongs to C^3 and $u_1(x, y)$ belongs to C^2.

Here we do not solve this problem, but we obtain an a priori estimate for the solution of this problem assuming $u_0(x, y)$ and