J. Math. Kyoto Univ.
2-1 (1962) 11-23.

On the Riemann's relation on open Riemann surfaces

By

Akira KOBORI and Yoshikazu SAINOUCHI

(Received August 1, 1962)

---

1. F being an arbitrary open Riemann surface, we consider an exhaustion $\{F_n\}$ ($n=1, 2, \cdots$) of F by regular regions satisfying the conditions:

i) for each n, F_n is a domain in F whose boundary Γ_n consists of a finite number of closed analytic curves in F,

ii) for each n, $\bar{F}_n = F_n \cup \Gamma_n \subset F_{n+1}$,

iii) $\bigcup_{n=1}^{\infty} F_n = F$

and

iv) for each n, any connected component of $F - F_n$ is non-compact in F.

Then there exists a canonical homology basis $A_1, B_1, \cdots, A_{k(n)}, B_{k(n)}, \cdots$ such that $A_1, B_1, \cdots, A_{k(n)}, B_{k(n)}$ form a canonical homology basis of F_n (mod ∂F_n) and $A_i \times B_j = \delta_{ij}$, $A_i \times A_j = B_i \times B_j = 0$[1)] (Ahlfors [1], Ahlfors-Sario [2]).

We denote by Γ_h the class of all square integrable harmonic differentials defined on F. The relation which expresses the inner product (ω, σ^*) for two differentials ω, $\sigma \in \Gamma_h$ (or subclass of Γ_h) in terms of periods of ω, σ is called the *Riemann's bilinear relation*, where σ^* denotes the conjugate differential to σ. Some conditions which insure the validity of the Riemann's bilinear relation are found by some suthors (Ahlfors [1], Pfruger [3], [4], Kusunoki

1) We note, throughout this paper, the intersection number of two cycles A, B is taken such that $A \times B$ has the positive sign when A crosses B from right to left as in [2]. Hence it has the opposite sign to that in [1].