Imbedding of an abstract variety in a complete variety

By

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(Received July 5, 1962)

The purpose of the present paper is to prove that an arbitrary abstract variety can be imbedded in a complete variety as an open set.

As for the terminology, we shall employ the one in the sequence of papers of ours in the American Journal of Mathematics ([2] (I, II, III)). We note that we need not assume that a ground ring is a Dedekind domain. Namely, our proof is valid without any modification in the case of models over a Noetherian integral domain, models being adapted to the case. Therefore the ground ring can be replaced also by a so-called Noetherian scheme, provided that every localities are integral domains.

In § 1, we state some of known theorems on birational correspondences. In § 2, we discuss a special kind of birational transformation, called dilatation. In § 3, we give some auxiliary results and in § 4 we give the proof of our main theorem.

The writer likes to add here that there has been one contribution by J. Ohm [4] to this problem saying that if V is an abstract variety, C is a curve on V and if there is a quasiprojective open covering $\{U_i\}$ of V such that C meets all the U_i , then there is an abstract variety V' containing V as an open subset in such a way that the closure of C in V' is a complete variety.

1. Birational correspondences.

We consider from now on only models whose function fields

¹⁾ The work was supported by NSF grant G14736.