

## Some results on Hausdorff $m$ -adic modules and $m$ -adic differentials

Dedicated to Prof. Akizuki for commemoration of his 60-th birthday

By

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**Introduction.** This paper consists of two parts. In Part I, we study some properties of Hausdorff  $m$ -adic modules and we introduce the notions of  $m$ -adic free modules (see, Definition 1). We see in §2 that  $m$ -adic free modules have very similar properties as free modules, especially when coefficients rings are local rings. In §4, we see that  $m$ -adic free modules are nothing but free modules, so far as only semi-finite modules (see, Definition 2) over local rings are considered (Theorem 2). Making use of the results in Part I, we develop in Part II the theory of  $m$ -adic differentials which has been introduced in [9]. The main results are as follows.

(1) We see in §2 that the notion of the  $m$ -adic differentials coincides with the notion of usual differentials, so far as we treat only localities defined over fields.

(2) In §3, we seek conditions for a prime ideal in an  $m$ -adic ring to be unramified over its subring, in terms of  $m$ -adic differentials. This is an analogous subject to that in Nakai [8], §5 or Kunz [4], §3.

(3) Regular local rings are characterized in our languages (§4 and §7). These are generalizations of a number of results in [9], §6 and §7. These are also generalizations of Satz 1 in Kunz [5] and Kunz's Satz 1 itself is proved by a different way. This shows that the theory of  $m$ -adic differentials is useful even in order to study the usual theory of differentials, like the fact that the method