

Note on paracompactness

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In this note, we shall be concerned with the paracompactness of a subspace of a Tychonoff space (completely regular T_1 -space). Generalizing the result previously given in [8, Th. 2.8], we shall give some necessary and sufficient conditions for a subspace of a Tychonoff space to be paracompact (Theorem 1). As a consequence of this, we shall obtain a new characterization of paracompactness (Theorem 2). In §2, we shall apply our theorem to a subspace $X \times M$ of $BX \times M$, where BX is a compactification of a paracompact space X and M is a metrizable space, and discuss the paracompactness of the product $X \times M$ (Theorem 3).

Theorem 3 in §2 stated originally that the product $X \times M$ of a hereditarily paracompact space X and a metrizable space M is paracompact if and only if it is normal and countably paracompact. The author is indebted to Prof. K. Morita for valuable remarks, in revision of Theorem 3.¹⁾

§1. PARACOMPACTNESS OF SUBSPACES

All spaces mentioned in this note will be completely regular and T_1 and all neighborhood will be assumed to be open. Let X

1) In his letter to the author, Prof. K. Morita has recently informed that he obtained the following result, which is slightly weaker than ours, with a theorem giving a necessary and sufficient condition for the product $X \times M$ of a normal space X and a metrizable space M to be normal and countably paracompact (to appear): The product $X \times Y$ of a paracompact space X and a metrizable space M is paracompact if and only if it is normal and countably paracompact. The author expresses his sincere thanks to Prof. K. Morita for his kindness.