J. Math. Kyoto Univ.
3-1 (1963) 127-135.

Notes on invariant differentials on abelian varieties

To Professor Y. Akizuki for the celebration of his 60th birthday

By

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(Communicated by Prof. Nagata, Aug. 26, 1963)

In our previous paper ([2])[1], we have proved the following: Let A be an abelian variety of dimension 2 and Γ a curve on A which generates A, and let ι be the injection of Γ into A. Then the adjoint map ι^* is a monomorphism. In this short note we shall give a generalization of the above result to the case of 3-dimensional abelian variety and a non-singular curve Γ on it which generates entire abelian variety. The method employed here is quite different from the former one and geometric in its nature. As a consequence we get an interesting result that if an abelian variety A of dimension ≤ 3 is generated by a non-singular curve Γ, then Γ generates A separably[2]. At the end we shall present related problems which are of some interest.

§1. Generalities on local derivations and tangent vectors

1. 1. We shall fix once for all a universal domain $\boldsymbol{K}$ in our algebraic geometry. Let X be a variety and let x be a point on X. We shall denote as usual by $\mathcal{O}_x$ the local ring of x in $\boldsymbol{K}(X)$ (=the function field of X). A *local derivation at* x is a derivation of $\mathcal{O}_x$ into $\boldsymbol{K}$ and a *semi-local derivation* at x is a derivation D of $\mathcal{O}_x \to \mathcal{O}_x$. We shall denote by π_x the natural map $\mathcal{O}_x \to \mathcal{O}_x/\mathfrak{M}_x = \boldsymbol{K}_x (=\boldsymbol{K})$,

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1) Number in the bracket refers to the bibliography at the end of the paper.
2) In the case of dimension 2 it was not necessary to assume that Γ is a non-singular curve (Cf. [2]).