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A note on transitive permutation groups of degree $p=2q+1$, p and q being prime numbers

To Professor Y. Akizuki on the occasion of his 60th birthday

By

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1. Let $p \geqq 5$ be a prime number and let Ω be the set of symbols $1, \cdots, p$. Let $\mathfrak{G}$ be a nonsolvable transitive permutation group on Ω. Let $p_0(\mathfrak{G})$ be the number of irreducible characters of $\mathfrak{G}$ whose degrees are divisible by p. It seems to be little known about the number $p_0(\mathfrak{G})$. In (**9**) it is shown that $p_0(\mathfrak{G})>0$. There exist a few groups with $p_0(\mathfrak{G})=1$; namely, $LF_2(l)$ as a permutation group of degree l ($l=5, 7, 11$), where $LF_2(l)$ denotes the linear fractional group over the field of 1 elements ((**2**), p. 286). In the present note, under the special condition that $\frac{1}{2}(p-1)=q$ is also a prime, we show that the converse of this fact holds; namely, we prove the following

Theorem. *Let* $q=\frac{1}{2}(p-1)$ *be also a prime. If* $p_0(\mathfrak{G})=1$, *then* $p=5, 7, 11$ *and* $\mathfrak{G}$ *is isomorphic to* $LF_2(p)$.

2. Throughout this section we assume that $q=\frac{1}{2}(p-1)$ is a prime. Then in (**6**), (**7**) and (**8**) we studied the structure of $\mathfrak{G}$ to some extent. In particular, we proved that such a group $\mathfrak{G}$ is triply transitive on Ω with the exception of $LF_2(7)$ and $LF_2(11)$. Now let us consider two irreducible characters $X_0(X)=\frac{1}{2}(\alpha(X)-1)(\alpha(X)-2)-\beta(X)$ and $X_{00}(X)=\frac{1}{2}\alpha(X)(\alpha(X)-3)+\beta(X)$ of the symmetric group on Ω, where $\alpha(X)$ and $\beta(X)$ respectively denote the the numbers of fixed symbols and the transpositions in the cycle