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A generalization of the imbedding problem of an abstract variety in a complete variety

To Professor Y. Akizuki for celebration of his 60th birthday

By

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In a previous paper [4], we proved that every abstract variety is an open subset of a complete abstract variety. In the present paper, we try to generalize this result to the case of a Neotherian scheme of finite type[1]. Namely, we consider first a ground Neotherian scheme S which is covered by a finite number of open Neotherian affine schemes S_i. Then a scheme we like to say to be of finite type over S is a scheme M over S such that M is covered by a finite number of open affine schemes M_j so that for a suitable choice of S_i, the morphism $M \rightarrow S$ induces morphisms $M_j \rightarrow S_i$ and the affine ring $\mathfrak{o}_j$ of M_j is finitely generated over the natural image of the affine ring of S_i in $\mathfrak{o}_j$.

Our main theorems imply that:

If M is a Noetherian scheme of finite type over a Noetherian ground scheme S, then M is an open subset of a proper scheme (Noetherian and of finite type) over S.

In our treatment, we use valuation-theoretic method, hence the usual definition of a scheme is not nicely suited to our proof. Therefore we give a valuation-theoretic definition of a Neotherian scheme of finite type. Then our method in our paper [4] can be adapted and we have our main results.

As for terminology on rings, we shall use mainly the one in our book [3].

1) As for the definition of a scheme, see Grothendieck [1].