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A. Skorohod's stochastic integral equation for a reflecting barrier diffusion

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1. Introduction. Given a standard Brownian sample path b with $b(0)=0$, let $[\mathfrak{x}, \mathfrak{t}]$ be a solution of

1a) $$\mathfrak{x}(t) = a+\int_0^t \sqrt{c_2[\mathfrak{x}(s)]}\,db+\int_0^t c_1[\mathfrak{x}(s)]\,ds+\mathfrak{t}(t)$$

1b) $$\mathfrak{x}(0) = a \geq 0^{2}$$

subject to the conditions: a) for each $t\geq 0$, $\mathfrak{x}(t)$ and $\mathfrak{t}(t)$ are Borel functions of the path $b(s): s\leq t$, b) $\mathfrak{x}(t)$ is continuous and non-negative, and c) $\mathfrak{t}(t)$ is continuous, non-negative, increasing, flat outside $\mathfrak{Z}=(t: \mathfrak{x}=0)$, and $\mathfrak{t}(0)=0$. A. SKOROHOD [6] proved that if $c_2>0$ and if

2) $$\begin{aligned}&|c_1(b)-c_1(a)| + |\sqrt{c_2(b)}-\sqrt{c_2(a)}| \\ &\leq \text{constant} \times (b-a) \qquad (0\leq a< b),\end{aligned}$$

then 1) has a unique solution $[\mathfrak{x}, \mathfrak{t}]$ for all but a negligeable class of Brownian paths, $\mathfrak{x}$ being identical in law to the diffusion with generator $\mathfrak{G}u=(c_2/2)u''+c_1u'$ subject to the reflecting barrier condition $u^+(0)=\lim_{\varepsilon\downarrow 0} \varepsilon^{-1}[u(\varepsilon)-u(0)]=0$.

SKOROHOD seems to have overlooked it, but $\mathfrak{t}$ is the local time[3] of $\mathfrak{x}$:

3) $$\mathfrak{t}(s) = c_2(0) \lim_{\varepsilon\downarrow 0} (2\varepsilon^{-1}) \text{ measure } (s: \mathfrak{x}(s)<\varepsilon,\ s\leq t),$$

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[2] $\int f\,db$ is an Itô stochastic integral.

[3] See K. Itô and H. P. McKean, Jr. [4] for information about local times.