On the imbedding of spherically symmetric space times

Dedicated to Professor Y. Akizuki on his 60th birthday

By

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§1. Introduction

The concept of a *spherically symmetric space time* is important in the general theory of relativity. Such a space time (abbreviated as *s.s. space* in this paper) is, mathematically, a Riemannian space whose fundamental form is reducible to

$$ds^{2} = -Adr^{2} - B(d\theta^{2} + \sin^{2}\theta d\varphi^{2}) + Cdt^{2},$$

$$B = r^{2} \quad \text{or} \quad \text{const.} > 0,$$
(1.1)

where r, θ , φ and t are spherical polar and temporal coordinates and A and C are positive-valued functions of r and t. An s.s. space is denoted by $S_{\rm I}$ or $S_{\rm II}$ according to whether B is r^2 or constant.

Many authors have investigated the geometrical properties of an s.s. space [1], among which the imbedding¹⁾ problem was discussed by Eiesland [2] and Takeno [3] along the line of the general theory of Riemannian spaces (see, for example, [4]). For application, however, it will be desirable to obtain a concrete picture of an imbedding. For the Schwarzschild space time, such a picture was used in Fronsdal's physical paper [5], and a related mathematical theory was developed by Fujitani and two of the present authors (M.I. & M.M.) [6]. This theory is extended to an s.s. space in the present paper.

¹⁾ By this we mean, throughout the present paper, the imbedding into a pseudo-Euclidean space.