J. Math. Kyoto Univ.
3-2 (1964) 251-273.

Homotopy groups of symplectic groups

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(Received Jan. 17, 1964)

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§1. Introduction

The present paper is one of our series on the homotopy groups of simple Lie groups, following from the previous paper [7].

We shall consider the homotopy groups $\pi_i(Sp(n))$ of symplectic groups $Sp(n)$.

When $i \leq 4n+1$, the groups are stable and computed by Bott [3]:

$$\pi_i(Sp(n)) \cong \begin{cases} Z & i \equiv 3, 7 \pmod 8, \quad i \leq 4n+1, \\ Z_2 & i \equiv 4, 5 \pmod 8, \quad i \leq 4n+1, \\ 0 & i \equiv 0, 1, 2, 6 \pmod 8, \quad i \leq 4n+1. \end{cases}$$

For almost stable cases $i=4n+2$, $4n+3$, $4n+4$, the following results are obtained (Theorem 2.2).

$$\pi_{4n+2}(Sp(n)) \cong \begin{cases} Z_{2\cdot(2n+1)!} & \text{for odd } n, \\ Z_{(2n+1)!} & \text{for even } n, \end{cases}$$

$$\pi_{4n+3}(Sp(n)) \cong Z_2,$$

$$\pi_{4n+4}(Sp(n)) \cong \begin{cases} Z_2 & \text{for odd } n, \\ Z_2 \oplus Z_2 & \text{for even } n. \end{cases}$$

These results were already computed in [4], except the last one which will be determined in §2 by use of secondary compositions.

For $i \leq 23$, the groups $\pi_i(Sp(1)) = \pi_i(S^3)$ and $\pi_i(Sp(2))$ are determined in [11], [6] and [7]. Then the following table of $\pi_i(Sp(n))$ is established by the computation of the groups $\pi_i(Sp(3))$, $17 \leq i \leq 23$, and $\pi_i(Sp(4))$, $21 \leq i \leq 24$. The computation will be given in §3