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Canonical conformal mappings of open Riemann surfaces

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Introduction

Koebe was the first who proved strictly the existence of conformal mappings of an arbitrary planar Riemann surface onto slit regions. Concerning an open Riemann surface R of positive genus g, Kusunoki [9] and Nehari [13] showed that for any $g+1$ points P_0, $P_1,\cdots,P_g$ on R there exists a conformal mapping of R with possible poles at $P_0,P_1,\cdots,P_g$ onto a covering surface of the extended plane which is at most $(g+1)$-sheeted and bounded by parallel slits, only if the boundary of R consists of a finite number of closed Jordan courves. Under the same condition on the boundary, Mori [12] showed the existence of conformal mappings of R onto covering surfaces of the extended plane which are $(g+1)$-sheeted and bounded by slits along parallel segments. As for circular and radial slit mappings, Kusunoki [10] and Nehari [13] established an analogous theorem under the same condition on the boundary. But the condition on the boundary is very restrictive, and our intention in this paper is to remove the restriction.

In §1 we shall consider some definitions and properties which are necessary for our conclusions. Here we also show that a Riemann surface of genus $g>1$ can be considered as an at most g-sheeted covering surface of the extended plane.

In §2 we shall treat parallel slit mappings. If f is a function on R which maps this onto a covering surface of the extended plane with q-sheets and parallel slits as a boundary, a subset E