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Note on semi-reductive groups

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The notion of a semi-reductive group was introduced in the preceding paper. The present note contains two results on semi-reductive groups.

The first result is as follows: Let G be a semi-reductive group contained in $GL(n, K)$, K being a field of characteristic p (which may be zero). Let ρ be a rational representation of G of type $\begin{pmatrix} 1 & \tau \\ 0 & \rho' \end{pmatrix}$, ρ' being a representation of degree one less than the degree m of ρ. We consider the action of G defined by ρ on the polynomial ring P_m in indeterminates $X_1, \cdots, X_m$ over K. Let $\mathfrak{a}$ be a G-stable ideal in P_m such that $\Sigma X_i K \cap \mathfrak{a} = 0$ and let x_i be the class of X_i modulo $\mathfrak{a}$. The semi-reductivity of G implies the existence of a G-invariant f in $K[x_1, \cdots, x_m]$ such that f is monic and of positive degree in x_1. Now the result is:

Theorem 1. *If there is such an f of degree d in x_1 so that d is not a multiple of p and if x_1 is transcendental over $K[x_2, \cdots, x_m]$, then ρ is equivalent to $\begin{pmatrix} 1 & 0 \\ 0 & \rho' \end{pmatrix}$.*

The other result concerns with the case of algebraic linear group, and can be stated as follows:

Theorem 2. *If an algebraic linear group G is semi-reductive, then the radical of G is a torus.*

We shall note in this article that any one of these theorems implies the following fact:

Proposition. *Let K be a field of characteristic zero and let G be a subgroup of $GL(n, K)$. Then G is reductive if and only if G is semi-reductive.*