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Invariants of a group in an affine ring

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With an Appendix
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1. When a group G acts on a ring R inducing a group of automophisms, then we can speak of G-invariants in R. Let us denote the set of G-invariants in R by $I_G(R)$. Our particular interest lies in the case where R is a finitely generated (commutative) ring over a field K and the action of G on R is such that 1) the automorphisms are K-isomorphisms and 2) $\Sigma_{g\in G} f^g K$ is a finite K-module for every $f\in R$. In this case, let $f_1, \cdots, f'_{n'}$ be a set of generators of R over K and choose a linearly independent base $f_1, \cdots, f_n$ of $\Sigma_i(\Sigma_{g\in G}(f_i)^g K)$. Then $R=K[f_1, \cdots, f_n]$ and the action of F on R is characterized by the representation of G defined by the module $\Sigma_{i,g} f_i^g K$. Thus, in order to observe $I_G(R)$, we may assume that

(1) G is a matric group contained in $GL(n, K)$, and

(2) $R=K[f_1, \cdots, f_n]$ and, for every $g\in G$, the automorphis of R defined by g is induced by the linear transformation

$$\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} \to g \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}.$$

Under the circumstance, the following results are known:

Lemma 1. *$I_G(R)$ is finitely generated if every rational representation of G is completely reducible or if G is a finite group, hence if G has a normal subgroup N of finite index such that every rational representation of N is completely reducible.*