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On the automorphisms of hypersurfaces*

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Introduction. When V is a projective variety, we denote by $\mathrm{Bir}(V)$ the group of birational transformations of V onto itself, by $\mathrm{Aut}(V)$ the group of automorphisms of V (i.e. the group of the biregular transformations of V onto itself), and by $\mathrm{Lin}(V)$ the subgroup of $\mathrm{Aut}(V)$ consisting of the elements induced by the projective transformations of the ambient space which leave V invariant. The last one is obviously an algebraic group, while $\mathrm{Aut}(V)$ has the structure of an "algebraic group with (eventually) countably-infinite number of components".

Let $H_{n,d}$ denote a hypersurface of degree d in the $(n+1)$-dimensional projective space $\boldsymbol{P}_{n+1}$, defined by an equation $f(X_0, X_1, \cdots, X_{n+1})=0$ of degree d. The main results of this memoir are:

(1) *If $H_{n,d}$ is non-singular and $n\geqslant 2$, $d\geqslant 3$, then $\mathrm{Aut}(H_{n,d})$ is finite except the case $n=2$, $d=4$.*

(2) *If $H_{n,d}$ is generic over the prime field and if $n\geqslant 2$, $d\geqslant 3$, then $\mathrm{Aut}(H_{n,d})$ is trivial except the following case: the ground field has characteristic $p>0$ and $n=2$, $d=4$.*

The exception in (1) is a real one, while in (2) it is likely that the theorem holds without exception, though we have to leave the question open. The main part of the proofs consists in showing that $\mathrm{Lin}(H_{n,d})$ is small. For the sake of completeness we have added a few known results.

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