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Atomic Quasi-Injective Modules

By

Edward T. WONG*

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1. Quasi-injective modules. R is an associative ring with an indentity element, all R-modules considered here are left unitary modules.

Given an R-module M and submodules N_1, N_2 of M, N_2 is called an essential extension of N_1 or N_1 is essential in N_2 if

(1) $N_1 \subset N_2$ and (2) $Rx \cap N_1 \neq 0$ for every nonzero x in N_2.

Condition (2) is equivalent to that every nonzero submodule in N_2 has nonzero intersection with N_1. If N_2 is an essential extension of N_1 then the left ideal $(N_1 : x) = \{r \in R \mid rx \in N_1\}$ is essential in R for every x in N_2 (considering R as a left R-module).

Every submodule of M has a maximal essential extension in M. If the singular submodule $M^{\blacktriangle}$ of M is zero then for every submodule N, N^s is the unique maximal essential extension of N in M, where

$$N^s = \{m \in M \mid (N : m) \text{ is essential in } R\}$$
$$M^{\blacktriangle} = \{m \in M \mid (0 : m) \text{ is essential in } R\}.$$

The verification of N^s being the unique maximal essential extension of N in M in the case of $M^{\blacktriangle} = 0$ is straight forward.

In this paper we assume every R-module has zero singular submodule unless stated otherwise. Of course if N_1 is essential in N_2 then $N_1^{\blacktriangle} = 0$ if and only if $N_2^{\blacktriangle} = 0$.

In [2] we call an R-module M quasi-injective if for any submodule N of M and any $f \in \mathrm{Hom}_R(N, M)$, f can be extended to an element of $\mathrm{Hom}_R(M, M)$. An injective module of course is quasi-injective.

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* The author is a National Science Foundation Faculty Fellow of U. S. A., 1963-64.