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On orbit spaces by torus groups

By

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Let V be an affine variety with universal domain K and let T be a torus acting on V in the usual sense.

Consider the set U of points of V whose orbits are of maximal dimension. Then we can think of orbit space U/T, which may not be a variety in general but is a prescheme. For simplicity, we denote by V/T the orbit space U/T. Let R be the coordinate ring of V over K. Then T acts also on R. The set $I_T(R)$ of T-invariants in R is finitely generated over K, hence defines an affine variety W.

The main result of our present article is that V/T is covered by a finite number of projective varieties over W.[1)2)]

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1. Formulation of the result

Let V be an affine variety with coordinate ring $R=K[x_1, \cdots, x_n]$. A variety X is called a *projective variety over* V if there is a set of elements $u_0, \cdots, u_m$ of a field containing R such that X is covered by affine varieties X_i defined by $R[u_0/u_i, \cdots, u_m/u_i]$ $(i=0, 1, \cdots, m)$. If one u_i (hence every u_j which is not zero) is transcendental over the function field of X then $R[u_0, \cdots, u_m]$ is called a *homogeneous coordinate ring* of X. $R[u_0, \cdots, u_m]$ is a graded ring in which (1)

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1) The definition will be recalled in §1 below.

2) Though we treat the case of usual varieties for the simplicity of formulation, this can be adapted easily to the case of affine schemes whose rings are finitely generated over K. The reason is that Theorem 2.1 in [2] can be adapted to the case.