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On Mumford conjecture concerning reducible rational representations of algebraic linear groups

By

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I. Let K be an algebraically closed field of an arbitrary characteristic and let G be an algebraic linear group. We consider only K-rational points of G. Let V be a vector space over K on which G acts as a group of K-automorphisms. We call V a rational G-module or G acts rationally on V, if for any element v in V the translates gv for all g in G generates only a finite dimensional subspace on which the induced action of G is a rational representation.

Definition 1. G is called semi-reductive if it has the following equivalent properties:

(1) For any exact sequence of finite dimensional rational G-modules

$$O \to W \to V \to K \to O$$

where K is considered to be a trivial rational G-module, there exists a positive integer m such that if we take the m-th symmetric products, the surjective G-homomorphism

$$V^m \to K^m \cong K$$

splits.

(2) Let $X_1, X_2, \cdots, X_n$ be indeterminates and suppose G acts rationally on the polynomial ring $K[X_1, X_2, \cdots, X_n]$ as a group of automorphisms of K-algebra in such a way that the K-subspaces $KX_1 + KX_2 + \cdots + KX_n$ and $KX_2 + KX_3 + \cdots + KX_n$ are G-stable and that X_1 is G-invariant modulo $KX_2 + KX_3 + \cdots + KX_n$. Then there exists a G-invariant homogeneous polynomial in $K[X_1, X_2, \cdots, X_n]$ which is monic in X_1.