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On the generalized Hopf homomorphism and the higher composition, Part I

Dedicated to Prof. A. Kobori for his 60th birthday

By

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§ 1. Introduction

Much progress has been made in the study of homotopy groups of spheres since H. Freudenthal defined the "Suspension" in his paper "Über die Klassen der Sphärenabbildungen", Composito. Math. 5 (1937), 299-314. Though many topologists have studied to compute the homotopy groups of spheres, the problem is still open.

The present paper attempts to define and study new generalized Hopf homomorphisms $\bar{H}_k: \pi_{i+1}(S^{n+1}) \to \pi_{i+1}(S^{kn+1})$ $(k=1, 2, \cdots)$. The difference among the miscellaneous Hopf homomorphisms will be studied. The higher composition than the secondary composition will also be constructed and its properties will be stated in this paper.

Sections 2 and 3 of the paper are devoted to the contruction of higher compositions. Their properties are stated in these sections. Main tool is the secondary composition.

In Section 4 we will give the formula of the Hopf homomorphism H defined in [10] for the higher composition.

In Section 5 we define the generalized Hopf homomorphism by use of the structure of the suspension space of the reduced product complex.

Section 6 is the application of Section 2 and the preparation for the forthcoming paper [11].

In [11] the author will compute the $(n+i)$-th homotopy groups