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A simple construction of certain diffusion processes

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1. The construction

The purpose of this note is to discuss what I believe to be the simplest rigorous approach to one-dimensional diffusion theory. The method used is not really new; the idea appears in some of the older physically-oriented literature, although usually in a mathematically incomplete form, and the same basic approach has been taken recently by McKean [10] in the case of Bessel processes. The possibilities of the method do not seem to have been widely appreciated, however. Although the class of diffusions which can easily be derived in this way is limited, the method does in these cases offer easy proofs of some rather deep results on the nature of sample functions.

We will obtain other diffusions from the standard Wiener (Brownian motion) process, which is here taken for granted, by a method quite reminiscent of K. Itô's approach via stochastic differential equations [9] (see also [3], chapter 9). However, the concept of a stochastic integral will not be needed. Let $m(y)$ be a real function which satisfies a Lipschitz condition of order 1 on the whole line, and consider the integral equation

$$y(t) = a + \int_0^t m[y(\tau)]\,d\tau + x(t)\,. \tag{1}$$

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