J. Math. Kyoto Univ.
4-1 (1964) 149-159

# Extremal length and Kuramochi boundary

Dedicated to Professor A. Kobori on his 60th birthday

By

Tatsuo FUJI'I'E

(Received Sept. 8, 1964)

---

## 1. Introduction

The notion of extremal length was first introduced by Ahlfors and Beurling [1] and various equivalent or extended definitions of extremal length were considered by J. Hersch [4], J. Jenkins [5] and others. Let $\Gamma$ be a family of locally rectifiable curves given on a domain $D$ of a Riemann surface $R$, and $\rho(z)|dz|$ be a conformal metric on $D$ with non-negative covariant $\rho(z)$ such that $\int_\gamma \rho|dz|$ is defined (possibly $\infty$) for all $\gamma \in \Gamma$. According to Jenkins, $\rho(z)$ will be said to be admissible for the problem of extremal length of $\Gamma$ (or briefly "admissible") when $A(\rho)=\iint_D \rho^2(z)dx\,dy \leqq 1$, and, putting $L_\rho(\Gamma)=\inf_{\gamma\in\Gamma}\int_\gamma \rho|dz|$ for each admissible $\rho(z)|dz|$, the extremal length of $\Gamma$ is defined as the square of $\sup_\rho L_\rho(\Gamma)$ when the supremum is taken over all admissible conformal metrics. This definition of extremal length is equivalent to that of Ahlfors-Beurling and that of Hersch when $\rho(z)$ is limited in the same class of measurability.

By this definition of extremal length we know at once that extremal length of the family $\Gamma_0$ of all locally rectifiable curves whose $\rho$-length are infinite for a certain admissible conformal metric $\rho(z)|dz|$ is infinite, because $\lambda(\Gamma)^{1/2}=\sup_\rho L_\rho(\Gamma_0) \geqq L_\rho(\Gamma_0)=\infty$.

Especially, if we take the family $\Gamma_1$ of divergent curves in $R-K$ ($K$: compact domain in $R$) which start from $\partial K$ and along