

A remark on square integrable analytic semiexact differentials on open Riemann surfaces

Dedicated to Professor A. Kobori on his 60th birthday

By

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1. For canonical homology basis $\{A_n, B_n\}_{n=1,2,\dots}$ on an open Riemann surface the necessary and sufficient conditions for the existence of a square integrable analytic semiexact differential with given A -periods were investigated by Virtanen [1], Kusunoki [2] and Sainouchi [3]. In this paper we shall give a condition for the uniqueness of the existence of such differentials, which contains my previous result in [3]. In part, we make use of the same method as that in the Ahlfors' proof (Ahlfors [4], Theorem 9) giving the condition which the surface should belong to the class O_{AD} .

2. Let \bar{W} be a compact bordered Riemann surface of genus g and $\{A_i, B_i\}_{i=1,2,\dots,g}$ be a cononical homology basis mod $\partial\bar{W}$. We denote by $\Gamma_{ase}(\bar{W})$ the class of analytic semiexact differentials defined on W and also denote by $\Gamma_{ase}^A(\bar{W})$ the subclass of $\Gamma_{ase}(\bar{W})$ such that all A -periods of its element vanish. For the compact bordered surface $\Gamma_{ase}^A(\bar{W}) \neq \{0\}$ and the period $\int_c \alpha$ ($\alpha \in \Gamma_{ase}^A(\bar{W})$) to any chain c in W is the bounded linear functional on $\Gamma_{ase}^A(\bar{W})$, hence there exists a unique differential $\varphi_0(c) \in \Gamma_{ase}^A(\bar{W})$ such that

$$(\alpha, \varphi_0(c)) = 2\pi \int_c \alpha$$

for all differentials $\alpha \in \Gamma_{ase}^A(\bar{W})$.