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# Remarks on the harmonic boundary of a plane domain

Dedicated to Professor A. Kobori on his 60th birthday

By

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**Introduction.** We denote by $R_F^*$ the compactification of an open disc $R$ and by $\Delta_F$ the harmonic boundary of $R$, here the compactification is the one studied in the former paper [5]. In the first chapter, we shall study the relation between $\Delta_F$ and the Martin minimal boundary of $R$ in connection with the harmonic measure on $R$. In the second chapter, we shall treat the multiply connected domain, and a certain theorem with respect to the cluster sets will be studied from the view point of the compactification.

**1. Martin minimal boundary $\Delta_1$ and $\Delta_F$.** Let $R$ be an open Riemann surface which admits the non-constant bounded harmonic functions, and let $\Delta_1$ be the Martin minimal boundary of $R$. At first, we shall treat some lemmas with respect to $\Delta_1$ to make use of them later on. These lemmas were given by C. Costantinescu and A. Cornea [1] in general case.

**Lemma 1.** *Let $D$ be a non-compact subregion of $R$ and $A$ be a subset of $\Delta_1$ such as $A=\{s\in\Delta_1; I_D K_S>0\}$, then it holds that*

$$1 = \int_A I_D K_s(p)d\chi(s)+\int_{\partial D} d\omega_p(\tilde{p})$$

*for any point $p$ in $D$.*

*Proof.* According to [1], $u=I_D u+H_D^u$ for any $u\in HP$. From this, we know that $K_s(p)=H_D^{K_s}(p)$ $(p\in D)$ for each $s\in\Delta_1-A$. In