

Contributions to the theory of differentials on open Riemann surfaces

Dedicated to Professor A. Kobori on his 60th birthday

By

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Introduction

Ahlfors [2] and Ahlfors-Sario [3] have extended the theory of differentials on open Riemann surfaces, and Kusunoki [6] have developed the theory of Abelian differentials on open Riemann surfaces. Accola [1] has established some results on bilinear relations with respect to those differentials. We shall here use the same notations for the classes of differentials as in Ahlfors-Sario [3], and discuss relations between those classes of differentials, including the bilinear relations.

In §1 we establish a relation between the class of canonical differentials and the class of distinguished differentials (Theorem 1), which asserts that a meromorphic differential φ is a canonical semiexact differential if and only if the real part of φ is distinguished. An essential tool used there is a method of principal operator investigated by Sario [10]. In §2 we generalize the notion of the finite bilinear relation, which was defined on Riemann surfaces of class O_{HD} by Accola [1], to arbitrary open Riemann surfaces, and extend the results obtained by Accola [1]. Analogous results have been obtained by Oikawa, but he does not yet publish them. Section 3 deals with three classes of Riemann surfaces; the class O_{KD} , the class of surfaces on which $\Gamma_{he} \cap \Gamma_{hse}^* \subset \Gamma_{he}^*$ is valid and the class of surfaces on which $\Gamma_{hm} = \Gamma_{he} \cap \Gamma_{ho}$ holds. We give