

# On stationary solutions of a stochastic differential equation\*

Dedicated to Professor A. Kobori on his sixtieth birthday

By

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## 1. Introduction

To begin with, let us introduce some preliminary notions.

Given a stochastic process  $X(t)$ ,  $-\infty < t < \infty$ ,  $\mathcal{B}_{uv}(X)$  denotes the least Borel algebra for which  $X(t)$  is measurable for every  $t \in [u, v]$ .

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