

# On the branching process for Brownian particles with an absorbing boundary

By

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(Received Dec. 24, 1964)

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## Introduction

Recently A. V. Skorohod [ 4 ] gave a general treatment of the branching process from a standpoint of the theory of Markov processes. In this paper we shall apply this to discuss some problems of a branching process which was studied by B. A. Sevast'yanov [ 3 ]. We shall discuss in particular the problem of the extinction and some limiting property of the number of particles. As for the latter our result corresponds to that of T. E. Harris [ 1 ] in the case of age dependent branching processes. In a recent book of Harris [ 2 ] this result was strengthened to the almost sure convergence but in our case it seems difficult to apply his arguments and we could not succeed in this point.

## § 1 Preliminaries

In general a branching process with particles of one type on a locally compact separable Hausdorff space  $S$  is determined if we are given a Markov process  $x_t(P_x, x \in S)$  on  $S$  and a system of branching measures  $(p_n(x), \Pi_n(x, dy))_{n=0}^{\infty}$  where  $p_n(x)$ ,  $x \in S$  satisfies

$$0 \leq p_n(x) \leq 1, \quad \sum_{n=0}^{\infty} p_n(x) = 1$$

and  $\Pi_n(x, dy)$ ,  $x \in S$ ,  $y = (y_1, y_2, \dots, y_n) \in S^n$  is a probability measure

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1) Independently a quite similar idea was given by K. Ito at the seminar of probability theory at Kyoto University.