

Analytic manifolds admitting parallel fields of complex planes

By

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In this paper we discuss an n -dimensional analytic manifold¹⁾ M^n admitting a field of complex r -planes which is parallel with respect to a given affine connection and has only the zero vector in common with its complex conjugate plane field.

In the case where $n=2r$, we have the theorem due to Patterson [2], that, *if a Riemann manifold M^{2r} admits a field of r -planes which is null and parallel with respect to a given positive definite metric g , the M^{2r} admits a complex analytic structure in terms of which g is a Kähler metric.* On the other hand, in the previous paper [1], we proved the theorem that, *if a Riemann manifold M^{2r+1} admits a field of r -planes satisfying the similar conditions, the M^{2r+1} admits an almost contact metric structure having the covariant constant φ -tensor.*

We will treat mainly the general case $r \leq \left[\frac{n}{2} \right]$. Recently K. Yano [5] introduced the notion of an f -structure including an almost complex structure and an almost contact structure. Our main result is that there is a close relation between an f -structure and the existence of a field of complex r -planes satisfying the above conditions.

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1) Throughout the paper we assume the manifolds and tensors, including vectors, to be of class C^∞ .