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Multi-dimensional diffusion and the Markov process on the boundary

Dedicated to the memory of the late Y. Taniyama

By

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W. Feller [5, 6, 7] determined all the diffusion processes in one dimension, since A. Kolmogorov introduced the diffusion equation in 1931. Stimulated by his work, which is of analytic character, followed the works of E. B. Dynkin, K. Ito, H. P. McKean, Jr., and D. Ray, and completely determined one dimensional diffusion in a satisfactory correspondence between probabilistic and analytic properties. The study of the Brownian motion by P. Lévy and the rigorous set-ups for probabilistic treatment by J. L. Doob seem to have had prepared a necessary background for these works.

Approaches to such a solution have been tried in the case of multi-dimensional diffusion on the basis of these researches, though it seems to be far from completion in any sense. A. D. Wentzell [36] tried to find all the diffusions determined by the equation of type

$$(0.1) \qquad \frac{\partial u}{\partial t}(t, x) = Au(t, x), \quad x \in D, \ t \in [0, \infty),$$

where D is a domain in a sufficiently smooth manifold of N dimensions[1] and A is an elliptic operator on $\overline{D}$, both D and A having sufficient regularities. He proved that any sufficiently smooth function

1) Wentzell assumed that D is a domain in the N-dimensional Euclidean space R^N. But the same treatment is possible in sufficiently smooth manifolds without any change.