J. Math. Kyoto Univ.
5-1 (1965) 87-142

# On iterated suspensions I.

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(Received October 21, 1965)

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## Introduction

The $(n+t)$-th homotopy groups $\pi_{n+t}(S^n)$ of $n$-spheres $S^n$ are stable if $n>t+1$ with respect to Freudenthal's suspension homomorphism $S:\pi_{n+t}(S^n)\to\pi_{n+t+1}(S^{n+1})$, and the $t$-stem group $\pi_t^S$ is the limit of $\{\pi_{n+t}(S^n)\}$.

Throughout this paper $p$ will denote an odd prime which is fixed. $\pi_i(X, A:p)$ indicates the $p$-primary component of $\pi_i(X, A)$. Serre [10] obtained the following direct sum decomposition:

$$\pi_{i+1}(S^{2m}:p) \approx \pi_i(S^{2m-1}:p) \oplus \pi_{i+1}(S^{4m-1}:p).$$

So, we shall devote to consider the groups $\pi_{2m-1+t}(S^{2m-1}:p)$ and $2k$-fold iterated suspensions

$$S^{2k}: \pi_{2m-1+t}(S^{2m-1}:p) \to \pi_{2(m+k)-1+t}(S^{2(m+k)-1}:p).$$

Moore [8] and Serre [10] proved that the above homomorphism $S^{2k}$ is an isomorphism if $t<2m(p-1)-2$, that is, the group $\pi_{2m-1+t}(S^{2m-1}:p)$ is stable if $m>(t+2)/2(p-1)$ and denoted by $(\pi_t^S:p)$.

The homomorphism $S^2$ is related with groups $\pi_i(\Omega^2S^{2m+1}, S^{2m-1})$ by the following exact sequence:

$$\cdots\to\pi_{i+1}(\Omega^2S^{2m+1}, S^{2m-1})\xrightarrow{\partial}\pi_i(S^{2m-1})\xrightarrow{S^2}\pi_{i+2}(S^{2m+1})$$
$$\xrightarrow{H^{(2)}}\pi_i(\Omega^2S^{2m+1}, S^{2m-1})\xrightarrow{\partial}\cdots.$$

In [13, Th. (8.3)] the author gave an exact sequence