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On formal rings

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Part I

1. In the following, we shall fix a ground field K of positive characteristic p.

Let R be an algebraic system composed by a system of sets of n-indeterminates, $x=(x_1, \cdots, x_n)$, $y=(y_1, \cdots, y_n)$, $\cdots$ (we call them generic points) and two sets of non-zero formal power series with coefficients in K

$$\varphi_i(x_1, \cdots, x_n;\ y_1, \cdots, y_n),\quad \psi_i(x_1, \cdots, x_n;\ y_1, \cdots, y_n),\quad 1 \leqq i \leqq n,$$

with respect to $2n$-indeterminates $x_1, \cdots, x_n;\ y_1, \cdots, y_n$, satisfying the following conditions;

(*F1*) *R is an abelian formal group with respect to* $(\varphi_1, \cdots, \varphi_n)$, (*see* [1]),

(*F2*) $\psi_i(\psi(x, y), z) = \psi_i(x, \psi(y, z))$, *we call* $(\psi_1, \cdots, \psi_n)$ *the multiplication of x and y in R,*

(*F3*) $\varphi_i(\psi(x, y), \psi(x, z)) = \psi_i(x, \varphi(y, z))$,

$\varphi_i(\psi(x, z), \psi(y, z)) = \psi_i(\varphi(x, y), z)$, $1 \leqq i \leqq n$.

We call R a *formal ring* of dimension n defined over K, and write

$$x \cdot y = (\psi_1(x, y), \cdots, \psi_n(x, y)),\quad x \dot{+} y = (\varphi_1(x, y), \cdots, \varphi_n(x, y)).$$