J. Math. Kyoto Univ.
5-2 (1966) 177-184

# Connected fields of arbitrary characteristic*

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(Communicated by Prof. M. Nagata)

(Received Jan. 10, 1966)

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The question of whether there exists a connected field of non-zero characteristic was posed to us by Paul Chernoff. The authors wish to thank Professor Nagata for several observations and suggestions which helped us to find this construction.

## §1. Adjoining an "arc of indeterminates"

Let $k$ be an arbitrary integral domain (without topology). Let $R$ be the polynomial ring gotten by adjoining to $k$ an independent indeterminate $T_\alpha$ for each $\alpha$ in the open interval $(0, 1)$. We also define $T_0=0$, $T_1=1$, these not being indeterminates, of course.

Given any $\varepsilon>0$, let us define $U_\varepsilon \subset R$ to be the union, over all finite sequences $\alpha_1, \cdots, \alpha_n$; $\beta_1, \cdots, \beta_n$ in $[0, 1]$ such that $\sum|\alpha_i-\beta_i|<\varepsilon$, of the ideals $(T_{\alpha_1}-T_{\beta_1}, \cdots, T_{\alpha_n}-T_{\beta_n})$. For $f\in R$, let us define $|f|=\inf_{f\in U_\varepsilon}\varepsilon$. Note that for all $f$, $f\in(T_1-T_0)$, hence $|f|\leqslant 1$.

Let us say that an $f\in R$ "involves" a $T_\gamma(\gamma\neq 0, 1)$ if $f\notin k[T_\alpha]_{\alpha\neq\gamma}$. We shall consider all $f$'s to involve $T_0$ and $T_1$.

We shall make use of the following fact, the verification of which we leave to the reader: Suppose $f\neq 0$ belongs to the ideal $(T_{\alpha_1}-T_{\beta_1}, \cdots, T_{\alpha_n}-T_{\beta_n})$. Then $f$ must involve two distinct elements

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* This work was done while the first author held National Science Foundation grant GP 2184, and the second author held an Intermediate Year National Science Foundation Graduate Fellowship.