Connections, metrics and almost complex structures of tangent bundles

By

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In a series of papers [3], ···, [7], a theory of connections of Finsler spaces has been developed from the standpoint of fibre bundles. Let $P(M, \pi, G)$ be the bundle of frames over a differentiable manifold M, and $B(M, \tau, F, G)$ be the tangent bundle over M. The Finsler connection is defined in the induced bundle Q from P by the projection τ of B, and many concepts about connections of Finsler spaces are generalized and systematically treated.

On the other hand, the differential geometry of tangent bundles has been studied by several authors. S. Sasaki [12], [13] introduced a Riemannian metric into tangent bundles of Riemannian manifolds in order to study the behavior of geodesics. K. Yano and E. T. Davies [16] generalized the notion to the case where the base manifold has a Finsler metric. T. Nagano [9] and P. Dombrowski [1] defined the natural almost complex and product structures on tangent bundles. Recently, K. Yano and A. J. Ledger [17], [18] investigated linear connections on tangent bundles and showed that it is possible to obtain some interesting connections from a connection on the base manifold.

The purpose of the present paper is to develop synthetically the differential geometry of tangent bundles from the viewpoint of Finsler connections. Thus, connections, metrics and other structures of tangent bundles may be regarded as a part of contents of the Finslerian geometry.

The terminology and signs of previous papers will be used