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Characterizations of canonical differentials

Dedicated to Prof. K. NOSHIRO on his 60th birthday

By

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Introduction

Under the systematical study of Abelian differentials on open Riemann surfaces Ahlfors introduced the notion of distinguished differentials and obtained Abel's theorem ([1∼3]). On the other hand, by generalizing the normalized potentials (R. Nevanlinna [10]) Kusunoki [6] defined the (semiexact) canonical differentials and developed the theory of Abelian integrals on open Riemann surfaces ([5∼8]). Meanwhile, M. Mori [9] pointed out that these two classes of differentials are essentially the same, more precisely, a distinguished (real) harmonic differential is the real part of a semiexact canonical differential and vice versa.

In the present paper we shall give further some characteristic properties of canonical differentials. First, in §2 we shall show the following characterization: let φ be a semiexact meromorphic differential on open Riemann surface R, then φ is a semiexact canonical differential if and only if (i) there is a compact set F on R such that $du = Re\,\varphi$ is exact on $R-F$ and $\|du\|_{R-F} < \infty$ (ii) for any regular compact region $K(\supset F)$ and any semiexact analytic differential $dU + i^*dU$ with a finite norm on $R-K$, we have

$$\langle du, dU\rangle_{R-K} = \int_{\partial K} u^*dU$$

This definition of canonical differentials is superficially quite different from the original one and that of distinguished differentials, in the sense that last two definitions express rather constructively the form of their elements. Moreover this enables us