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Note on formally projective modules*

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§ **1.** Let R be a commutative ring with units and $\mathfrak{m}$ an ideal of R. Let M be an R-module. For the simplicity we assume that $\mathfrak{m}$ has a finite base. We consider the $\mathfrak{m}$-adic topology on both R and M. A. Grothendieck introduced the notion of formally projective modules which can simply be stated in our case as follows (19.2, Chap. 0_{IV} [1]).

Definition 1: M is called a formally projective module if $M/\mathfrak{m}^n M$ is a projective $R/\mathfrak{m}^n$-module for every $n=1, 2, 3, \cdots$.

On the other hand the authur introduced the notion of $\mathfrak{m}$-adic free modules (Def. 1, 2, Part I, [2]), i.e.

Definition 2: M is called an $\mathfrak{m}$-adic free module if M is a Hausdorff $\mathfrak{m}$-adic module and contains a set of elements $\{\alpha_i\}_{i\in I}$ such that $M/\mathfrak{m}^n M$ is a free $R/\mathfrak{m}^n$-module with a free basis {the residue clase of a_i mod. $\mathfrak{m}^n M\}_{i\in I}$ for every $n=1, 2, 3, \cdots$. In this case we call $\{\alpha_i\}_{i\in I}$ $\mathfrak{m}$-adic free basis of M.

We introduce here a generalized notion of $\mathfrak{m}$-adic free modules.

Definition 3: M is called a weakely $\mathfrak{m}$-adic free module if

(a) $M/\mathfrak{m}^n M$ is a free $R/\mathfrak{m}^n$-module for every $n=1, 2, 3, \cdots$, or equivalently

(b) the $\mathfrak{m}$-adic completion of M is isomorphic to the $\mathfrak{m}$-adic completion of a free R-module.

As for the equivalence of (a) and (b), we shall see it afterwards. $\mathfrak{m}$-adic free modules are weakly $\mathfrak{m}$-adic free modules.

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