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On the principle of limiting amplitude for dissipative wave equations

By

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§1. Introduction. The main theorem

In this paper we shall give a proof of the so-called principle of limiting amplitude for the initial value problem

$$\left\{\frac{\partial^2}{\partial t^2}+b(x)\frac{\partial}{\partial t}-\Delta+c(x)\right\}u(x,t)=f(x)e^{i\omega t}, \tag{1.1}$$

$$u(x,t)\Big|_{t=0}=\frac{\partial u(x,t)}{\partial t}\Big|_{t=0}=0. \tag{1.2}$$

Here x denotes a point in the three dimensional euclidean space E_3, Δ denotes the Laplacian in E_3 and ω is a real number. We shall treat the case where $b(x)\geqslant 0$, i.e., $b(x)$ is a positive dumping factor, and the potential $c(x)$ is a real valued function. Under these assumptions, we call (1.1) the *dissipative wave equation*. The principle of limiting amplitude states that every solution of the above problem tends to the steady state

$$e^{i\omega t}v(x,i\omega)$$

uniformly on bounded sets as $t\to\infty$. Here $v(x,i\omega)$ satisfies the elliptic equation

$$\{-\Delta+c(x)+i\omega b(x)-\omega^2\}v(x,i\omega)=f(x)$$

with the Sommerfeld radiation condition at infinity. This principle has been formulated and justified by Ladyženskaja [5], Èidus [2], Morawetz [7] and others from various standpoints and by different methods. However they all treated the case when $b(x)\equiv 0$ and no