ε-entropy of subsets of the spaces of solutions of certain partial differential equations

by

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§1. Introduction

In recent years ε -entropy of subsets of various function spaces were estimated by A. N. Kolmogorov and B. M. Tikhomirov [2] and by others. In the present paper we estimate ε -entropy of sets in the space of harmonic functions (published earlier in [3]) and the space of solutions of certain parabolic equation.

Our results are stated in §2 after the definition of ε -entropy is stated. We prove two lemmas in §3 and the conditions of these lemmas are examined separately for each case in §4 and §5.

The author expresses his hearty thanks to Professor H. Yoshizawa who suggested (in 1962) the problem of estimating the ϵ entropy of sets in space of solutions of partial differential equations.

§2. Definitious and statement of the results

Following [2], we shall list definitions which are necessary to state our results.

Let R be a metric space and A a subset of R.

Difinition 1. A system γ of sets $U \subset R$ is called *e-covering* of A, if $A \subset \bigcup_{U \in \gamma} U$ and the diameter of each $U \in \gamma$ does not exceed 2ϵ .

Definition 2. A set B in R is called *e-separated* if the distance