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Rational sections and Chern classes of vector bundles*

by

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Let $\mathcal{E}$ be a quasi-coherent sheaf, of finite type, on an integral prescheme X, and denote by $\mathbf{V}(\mathcal{E})$, $\mathbf{P}(\mathcal{E})$ the vector and projective fibres of $\mathcal{E}$ respectively. Then each non-zero rational section ω of $\mathbf{V}(\mathcal{E})$ over X defines a rational section $\bar{\omega}$ of $\mathbf{P}(\mathcal{E})$ over X (section 2), and we can construct a closed subscheme $\langle\omega\rangle$ of X whose points are the non-regular points of $\bar{\omega}$ (Prop. 5). Denote by $[\omega]$ the X-prescheme obtained by blowing up centered at $\langle\omega\rangle$. On the other hand we can construct a quasi-coherent fractional Ideal $\mathcal{O}_X(\omega)$ of the sheaf of rational functions $\mathcal{R}(X)$ of X which is invertible when X is *UFD* (Cor. of Prop. 4) and which corresponds to the Cartier divisor of the rational section ω.

In this note, we shall prove some relations between these schemes or sheaves (Th. 1.2). In the case that X is a non-singular quasi-projective algebraic scheme, they give an explicite formula of Chern classes of vector bundles of rank 2 (Cor. of Th. 2′). And, as a special case, if X is a surface and $\mathbf{V}(\mathcal{E})$ is the bundles of simple differentials, then our formula proves that the Severi-series of X coincides with the second Chern class $c_2(X)$ of X (last Remark).

1. Rational maps and rational functions (EGA. I. 7) Let X and Y be S-preschemes, and $\mathfrak{U}_X$ the set of dense open subsets of X; then the family of sets of S-mophisms $(\mathrm{Hom}_S(U, Y))_{U\in\mathfrak{U}_X}$

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