A duality theorem for locally compact groups

By

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Summary

The purpose of the present paper is to prove a dual relation between locally compact groups G and the set Ω (resp. $\widehat{G}$) of equivalence classes of (resp. all irreducible) unitary representations of G. This duality may be considered as an extension of Pontrjagin duality for abelian groups and Tannaka duality for compact groups.

In such a duality theorem, G is characterized as the "dual group" of Ω (resp. $\widehat{G}$), that is, as the set of all "birepresentations" which are operator fields over Ω (resp. $\widehat{G}$) commuting with the operation of Kronecker product. "Birepresentation" is a generalization of a character over the dual group in abelian case. And the initial topology of G coincides with the "weak topology" on the set of operator fields over Ω.

The duality between G and Ω is called the weak duality, and the one between G and $\widehat{G}$ is called the strong duality. The first one is proved for general locally compact groups, but the strong duality is proved under the type I restriction for G.

The results are strengthened for special groups.

§0. Introduction.

1. In the theory of representations of locally compact groups, the dual object, that is, the set of all equivalence classes of unitary