J. Math. Kyoto Univ.
6-2 (1967) 177-185

On the jacobian varieties of the fields of elliptic modular functions II.

By

Koji Doi* and Hidehisa Naganuma

(Roceived November 29, 1966)

The purpose of this note is to observe the Galois groups of normal extensions obtained by the coordinates of the ideal section points of the jacobian variety J_q of an algebraic curve uniformized by elliptic modular functions, which was investigated in a previous work [2] with the same title. Our result can be obtained by slight modification of the consideration due to G. Shimura [6]. In fact, in his [6, footnote 9), p. 281], our problem was suggested.

In §4 of the present paper, we treated a simple jacobian variety J_q of dimension 2, having a real quadratic number field $\boldsymbol{Q}(\sqrt{d})$ as its endomorphism algebra. By a numerical example, we shall show that there occur two types of Galois group $G(K(\mathfrak{l})/\boldsymbol{Q})$, according as $\left(\frac{d}{l}\right)=+1$ or -1, which is isomorphic to $GL(2, GF(l))$ or $GF(l)^{*}\cdot SL(2, GF(l^2))$ respectively, where $\mathfrak{l}$ $(|l)$ denotes a prime ideal in $\boldsymbol{Q}$ $(\sqrt{d})$ and $K(\mathfrak{l})/\boldsymbol{Q}$ a normal extension generated by the coordinates of the $\mathfrak{l}$-section points of J_q.

Notations. Let F be an algebraic number field of finite degree over $\boldsymbol{Q}$ and $\mathfrak{o}$ be the ring of integers in F. Let (A^n, θ) be an abelian variety of type (F) in the sense of [4] i.e. a couple (A, θ) formed by an abelian variety A of the dimension n and an isomorphism θ of F into $\mathrm{End}_{\boldsymbol{Q}} A = \mathrm{End}\, A \otimes_{\boldsymbol{Z}} \boldsymbol{Q}$ such that $\theta(1) = 1_A$ (=the identy element of $\mathrm{End}_{\boldsymbol{Q}} A$). In the following treatment, (A^n, θ) will denote

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* This work was partially supported by The Sakkokai Foundation.