The Homotopy groups of Lie groups of low rank

By

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§1. Introduction

The compact simply connected simple Lie groups are classified as follows:

$$A_n = SU(n+1), \quad B_n = Spin(2n+1), \quad C_n = Sp(n), \quad D_n = Spin(2n)$$

$$G_2, \ F_4, \ E_6, \ E_7, \ E_8,$$

where $A_1 = B_1 = C_1$, that is, $SU(2) = Spin(3) = Sp(1)$,

$\quad\quad B_2 = C_2 \quad$, that is, $Spin(5) = Sp(2)$,

and $\quad A_3 = D_3 \quad$, that is, $SU(4) = Spin(6)$.

The first four types are called the classical Lie groups, and the last five are called the exceptional Lie groups.

The purpose of this paper is to determine the first 23 homotopy groups of G_2, F_4, and of B_n and D_n of low rank.

This paper is divided into two parts. The first part consists of §2 and §3. In §2 we calculate the cohomology groups of the 3-connective fibre space over G_2 and F_4. In §3, we compute the odd primary components of the homotopy groups of G_2 and F_4 by the killing-homotopy method [6].

We study in §4 some properties in the homotopy theory of the fibre spaces, especially, of the bundles. These are used in §6 for the determination of $\pi_i(G_2)$.

Section 5 is an intermediate one. It is the preparation for the second part, which consists of §6, §7, §8 and §9. In §6 we deter-