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Jacobson-Bourbaki Correspondence

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P is a field. Considering P as an abelian group with respect to its addition, let R be the ring of all endomorphisms of P into P. R is a right P-vector space in the obvious way. A subring $\mathfrak{A}$ of R containing the identity mapping is called a P-subring of R if $\mathfrak{A}$ is also a P-subspace of R. If $\mathfrak{A}$ is a P-subring of R, let $\Delta_{\mathfrak{A}}=\{\alpha\in R \mid \alpha A=A\alpha\}$, for all $A\in\mathfrak{A}$, the centralizer of $\mathfrak{A}$. $\Delta_{\mathfrak{A}}$ can be considered as a subfield of P. When P is considered as a left $\Delta_{\mathfrak{A}}$ vector space, $\mathfrak{A}$ is a dense ring of linear transformations of P. This follows from the fact that, $P\subset\mathfrak{A}$, $x\in P$, $x\neq 0$, $x\mathfrak{A}=P$; and the general density theorem [2]. The Jacobson-Bourbaki Theorem [1, p. 22] states that: "If the dimension of a P-subring $\mathfrak{A}$ over P is $n<\infty$, then the dimension of P over $\Delta_{\mathfrak{A}}$ is also n and $\mathfrak{A}=\mathfrak{L}_{\Delta_{\mathfrak{A}}}(P, P)$, the complete ring of linear transformations of P over $\Delta_{\mathfrak{A}}$." From this result, one can set up an one-to-one correspondence (Jacobson-Bourbaki Correspondence) between the set of all P-subrings of R which are finite dimensional over P and the set of all subfields of P which are finite co-dimensional in P [1, p. 24]. Furthermore the classical Galois theorem about finite group of automorphisms of a field can be obtained from this approach [1, p. 29].

In this paper, we are going to extend this correspondence further.

A ring S is called a *left* (*right*) *self-injective* ring if S is a left (right) injective module over itself. A left (right) self-injective ring is also called a *left* (*right*) *quasi-Frobenius* ring.

A ring Q is called a *left quotient ring* of a subring T of Q,