J. Math. Kyoto Univ.
6-3 (1967) 359-382

Behavior of the Solutions of Certain Heat Equations

By

Frank B. KNIGHT

(Communicated by Prof. H. Yoshizawa, November 22, 1966)

Introduction. We shall be concerned with equations which, in the simplest case, have the form

0. 1) $$\left(\frac{1}{2}\frac{\partial^2}{\partial x^2}+V(x)\right)g(t, x)=\frac{\partial}{\partial t}g(t, x), \qquad -\infty<x<\infty,$$

and denote the operator $\frac{1}{2}\frac{d^2}{dx^2}+V$ by A_V. Under weak conditions on V there corresponds to 0.1) a strongly continuous semigroup T_t on a suitable space $\tilde{C}$ of continous functions such that the function $T_tf(x)=\int\tilde{p}(t, x, y)f(y)dy$ for $f\in\tilde{C}$ is a solution of 0.1) and $\tilde{p}$ is uniquely determined by T_t and continuity in y. Our primary concern is with the shape and the evolution with t of the kernels $\tilde{p}$.

The kernel $\tilde{p}(t, x, y)$ can be considered as the temperature at time t and point y resulting from a unit source of heat at x when $t=0$. A more detailed interpretation is obtained, however, in terms of a family of measure spaces $(\Omega, \mathcal{F}, \mu_x)$ for which the elements of Ω are of the form (η, ρ, w), $0\leq\eta<\rho\leq\infty$, $w=w(t)$, $0\leq t<\rho$, where $w(t)$ is continuous and $T_tf(x)=E_x(f(w(t))\,;\,\eta<t<\rho)$ E_x denoting an integral with respect to μ_x. The functions $w(t)$ represent the paths of particles created at time η and destroyed at time ρ. These space provide, however, only one possible measure-theoretic approach to T_t. They are defined, for bounded,