J. Math. Kyoto Univ.
7-1 (1967) 71-75

On complete homogeneous surfaces

By

Makoto ISHIDA

(Communicated by Prof. M. Nagata, April 24, 1967)

It is known that a complete nonsingular curve C, which is a homogeneous space for a connected algebraic group, is birationally isomorphic to either an abelian variety of dimension 1 or the projective line $\boldsymbol{P}^1$.

The purpose of this paper is to prove a similar result for the two-dimensional case. That is, we shall give the proof of the following

Theorem. *Let F be a complete nonsingular surface, which is a homogeneous space for a connected algebraic group G. Then F is birationally isomorphic to one of the following:*

1) *an abelian variety A of dimension 2,*

2) *a bijective rational image*[1] *of the direct product $A\times\boldsymbol{P}^1$ of an abelian variety A of dimension 1 and the projective line $\boldsymbol{P}^1$,*

3) *the projective space $\boldsymbol{P}^2$ of dimension 2,*

4) *the two-fold direct product $\boldsymbol{P}^1\times\boldsymbol{P}^1$ of the projective line $\boldsymbol{P}^1$.*

Of course, if the characteristic of the universal domain is 0, then 2) is same to

2′) *the direct product $A\times\boldsymbol{P}^1$.*

We note that an algebraic homogeneous space can be embedded in some projective space (cf. [2]). Hence, in order to prove the theorem, we may assume that the complete homogeneous surface F is contained in a projective space.

1) This means that F is (birationally isomorphic to) the image of $A\times\boldsymbol{P}$ by a bijective regular rational mapping.