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Derivations in Azumaya algebras

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§ 1. Introduction

Let A be an Azumaya algebra, i. e. a central separable algebra, over a commutative ring R. A theorem of Jacobson-Hochschild states that if R is a field, then any derivation of R can be extended to a derivation of A ([6], Theorem 2); the proof consists in reducing the problem to crossed products and using a cohomological argument. We prove here that the theorem is valid, more generally, for any semi-local ring R; even for the case of a field is "functorial". The method of proof is as follows: in § 2 we show that for any commutative ring R, the canonical homomorphism $R[X]/(X^2) \to R$ induces an isomorphism of the corresponding Brauer groups; this comes out as a corollary to Theorem 2. 1, which seems to be of independent interest. In § 3 we prove the Skolem-Noether theorem over semi-local ring (Theorem 3. 1) and deduce a "cancellation law" for Azumaya algebras over such rings. We use these facts in § 4 to prove the main theorem.

For standard concepts and results regarding Azumaya algebras and Brauer groups over commutative rings, we refer to Auslander-Goldman [3] and Bass [4].

In what follows, R will denote a commutative noetherian ring and $\otimes$ will stand for $\otimes_R$.

§ 2. A theorem on Brauer groups

Theorem 2. 1. *Let R be a commutative noetherian ring and let $\mathfrak{A}$ be an ideal of R contained in the radical such that R is complete*