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On the unique factorization theorem for formal power series

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Let $R\{x_1, \cdots, x_n\}$ be the formal power series ring in a finite number of independent variables $x_1, \cdots, x_n$ with coefficient ring R. It is known that even if R is a unique factorization domain $R\{x_1\}$ is not always so.[1)]

We shall denote the following condition for a ring[2)] R by $(*)$:

$(*)$ $R\{x_1, \cdots, x_n\}$ *is a unique factorization domain, for any n (finite).*

It is noted that $(*)$ is satisfied by a regular semi-local integral domain R, which follows from the fact that a regular local ring is a unique factorization domain. This naturally raises the question whether the unique factorization theorem still holds for the case of infinitely many variables, provided coefficient domain R satisfies $(*)$. The question is only partially answered below (Theorem 1), where notion of formal power series is taken in a wider sense than the usual one.

As for the usual formal power series, what we show is that if R is a Krull ring then $R\{x_1, x_2, \cdots, x_n, \cdots\}$ is also a Krull ring, which is an application of Theorem 1.

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1. Let R be a ring, X be a set of indeterminates, card. $X = \aleph^*$. As usual, by a *X-monomial* $(x)^e$ *of degree* n ($n=0, 1, 2, \cdots$) we mean

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1) See P. Samuel, *Anneaux factoriels*, Publicações da Sociedade de Matemática de São Paulo, 1963, pp. 58-63.

2) A *ring* in this note always means a commutative ring with 1.