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Some remarks on boundary values of harmonic functions with finite Dirichlet integrals

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INTRODUCTION. In this article we shall show some results concerning the boundary values of an HD-function (a single-valued harmonic function with finite Dirichlet integral) given as the limit of HD-functions.

Let R be an open Riemann surface of hyperbolic type or more generally a Green space. We consider a D-normal compactification R^* of R, a notion introduced by Maeda [6], and the ideal boundary $\Delta = R^* - R$. Several well-known compactifications, for instance, those of Wiener, Royden, Martin and Kuramochi are D-normal. Every HD-function u on R is, by definition, expressed as $u(a) = \int_\Delta f d\omega_a$ with a resolutive function f and the harmonic measure ω. The f is determined except a set of harmonic measure zero and is denoted by $H^{-1}u$. In some compactifications $H^{-1}u$ is given as the limit values of u. We extend the definition of the linear operator H^{-1} to define $H^{-1}u$ for HD-functions u given outside of compact sets on R and prove Theorem 2 which will play a fundamental role in the sequel. From Theorem 3 we shall derive Theorem 4 which is regarded as a generalization of the corresponding theorem in Kusunoki [5] obtained for Kuramochi boundary.

1. In the following we shall denote by R an open Riemann surface of hyperbolic type. First of all we state the following known