

## PL-submanifolds and homology classes of a PL-manifold II

By

Masahisa ADACHI

(Received August 7, 1967)

---

In [1] we have proved the fundamental theorem of the realization problem of homology classes by submanifolds in the *PL*-case. In the present paper we shall give some consequences. One is based on the results of Browder-Liulevicius-Peterson [2] on the homotopy types of the *PL* Thom spectrum *MPL*, and the other on the results of Kuiper-Lashof [5] on the homotopy groups of *PL*<sub>1</sub>.

We use the notations and terminologies in [1].

We wish to thank Professors F. P. Peterson and H. Toda for their help in the preparation of this paper.

### 1. Statements of the results

**Theorem 1.** *Let  $V^n$  be a closed PL-manifold of dimension  $n$ . For  $k \leq n/2$ , all homology classes of  $H_k(V^n, Z_2)$  can be realized by PL-submanifolds which have normal PL-microbundles.*

**Theorem 2.** *Let  $V^n$  be a closed PL-manifold of dimension  $n$ . All homology classes of  $H_{n-1}(V^n, Z_2)$  can be realized by PL-submanifolds which have normal PL-microbundles.*

These results are quite parallel to those of  $C^\infty$ -case in Thom [8].

### 2. Study of the homotopy type of Thom complexes $MPL_k$

a) Preliminaries.

Let  $MPL = \{MPL_n, \mu_n; n \geq 0\}$  be the *PL* Thom spectrum defined