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# Regular operators and spaces of harmonic functions with finite Dirichlet integral on open Riemann surfaces

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**Introduction** Let $R$ be an open Riemann surface and let $W$ be an open subset of $R$ consisting of a finite number of regularly imbedded regions on $R$ such that $R-W$ is connected and compact. We denote by $C^{\omega}(\partial W)$ the family of real analytic functions on $\partial W$ and by $H(\overline{W})$ the family of harmonic functions on $\overline{W}$. L. Sario introduced (see [1]) the notion of a normal operator $L$: $C^{\omega}(\partial W)\rightarrow H(\overline{W})$ which is defined by the following conditions:

$$(1)\ \ Lf=f \text{ on } \partial W, \quad (2)\ \ L(c_1f_1+c_2f_2)=c_1Lf_1+c_2Lf_2,$$

$$(3)\ \ L1=1, \quad (4)\ \ Lf\geqq 0 \text{ if } f\geqq 0, \quad (5)\ \ \int_{\partial W}(dLf)^*=0.$$

One of Sario's important results is the following existence theorem for principal functions: *Let a harmonic function $s$ be given on $\overline{W}$. Then there exists a harmonic function $p$ on $R$ satisfying $p-s=L(p-s)$ on $W$ if and only if $\int_{\partial W}(ds)^*=0$. The function $p$ is uniquely determined up to an additive constant.*

He constructed two normal operators $L_0$ and $(P)L_1$. Using the above existence theorem for these operators he gave elegant proofs to some classical theorems and obtained some results which have been applied to the theory of conformal mapping by many authors ([6], [9], [10], etc). However neither Dirichlet operator $H^W$ ([3]