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## Branching Markov processes II

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The branching property of semi-groups and branching Markov processes were treated in Part I but the problem of construction was not discussed. We shall construct  $(X^0, \pi)$ -branching Markov processes in a probabilistic way. We shall first give a theorem on constructing a strong Markov process from a given Markov process by a piecing out procedure generalizing a method of Volkonsky [44], where a lemma on Markov time due to Courrège and Priouret [4] plays an important role. In chapter III, we shall apply the theorem to obtain  $(X^0, \pi)$ -branching Markov processes and give several examples.

The numbering continues that of the first part, pp. 237-278 of this journal. References such as [1] are to the list at the end of the first part.

## II. Construction of a Markov process by piecing out

## §2.1. Construction

Let *E* be a locally compact Hausdorff space with a countable open base,  $(W, \mathcal{B})$  be a measurable space on which a system  $\{P_x, x \in E\}$ of [probability measures is given, and  $\mu(w, dy)$  be a stochastic kernal on  $(W, \mathcal{B}) \times (E, \mathcal{B}(E))$ .<sup>1)</sup> Let  $\mathcal{Q} = W \times E$ ,  $\mathcal{F} = \mathcal{B} \otimes \mathcal{B}(E)$  and

<sup>1)</sup> We assume that, for every  $B \in \mathcal{B}$ ,  $P_x[B]$  is  $\mathcal{B}(E)$ -measurable in x. A stochastic kernel  $\mu(w, dy)$  is a kernel such that for each w it is a probability in dy.