

# The influence of small values of a holomorphic function on its maximum modulus

By

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## 1. Introduction

In a recent paper [5] we investigated the possible growth of the maximum modulus of a holomorphic function  $f$  defined in the unit disk  $D$  if the function tended to zero on certain sequences of Jordan arcs  $\{\gamma_n\}$  in  $D$ . These sequences were distinguished by having

$$(1.0) \quad \begin{aligned} \text{i)} \quad & \frac{1}{2} \leq r_n = \min_{z \in \gamma_n} |z| \rightarrow 1, \quad n \rightarrow \infty; \\ \text{ii)} \quad & 0 < \varliminf_{n \rightarrow \infty} HD(\gamma_n) \leq \overline{\lim}_{n \rightarrow \infty} HD(\gamma_n) < \infty; \end{aligned}$$

where  $HD(\gamma_n) = \sup \rho(a, b)$ ,  $a, b \in \gamma_n$ ,  $\rho(a, b)$  denoting the hyperbolic distance between  $a$  and  $b$ . Such a sequence satisfying (1.0) is labeled a *PHD* sequence. If

$$R_n = \max |z|, \quad z \in \gamma_n, \quad n = 1, 2, \dots,$$

then the closed circular sector of  $|z| \leq R_n$  of minimum angle  $\alpha_n$  containing  $\gamma_n$  is denoted by  $E_n$ . So  $E_n$  is of the form

$$0 \leq |z| \leq R_n, \quad \theta_n \leq \arg z \leq \theta_n + \alpha_n.$$

For convenience we suppose  $0 \leq \alpha_n \leq \pi$ , all  $n$ . For a *PHD* sequence

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